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SIMULATION OF HORIZONTAL PIPE TWO-PHASE SLUG FLOWS USING THE TWO-FLUID MODEL

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Resumo

O escoamento com padrão de golfadas ocorre em muitas aplicações de engenharia, especialmente no transporte de hidrocarbonetos ao longo de oleodutos. A intermitência do escoamento em golfadas causa um carregamento transiente severo nas tubulações levando a problemas de projeto. Portanto, é importante ser capaz de prever a formação e o desenvolvimento do escoamento no padrão de golfadas, assim como as características da golfada. O presente trabalho consiste na simulação do escoamento bifásicos no padrão de golfadas através de tubulações horizontais, utilizando o modelo de dois fluidos em sua forma transiente uni-dimensional. O presente modelo permite que o escoamento se desenvolva naturalmente no padrão de golfadas, a partir das condições iniciais. Para ilustrar a metodologia, simulações foram realizadas para diferentes situações que levam a formação do padrão de golfadas.

Abstract

Slug flow occurs in many engineering applications, mainly in the transport of hydrocarbon fluids in pipelines. The intermittency of slug flow causes severe unsteady loading on the pipelines carrying the fluids, which gives rise to design problems. Therefore, it is important to be able to predict the onset and development of slug flow as well as slug characteristics. The present work consists in the simulation of two-phase flow in slug pattern through horizontal pipes using the two-fluid model in its transient and one-dimensional form. The advantage of this model is that the flow field is allowed to develop naturally from a given initial conditions as part of the transient calculation; the slug evolves automatically as a product of the computed flow development. Simulations are then carried out for a large number of flow conditions that lead a slug flow.

1. Introduction

Two-phase flow in the slug pattern can be found in several engineering applications, such as flow of hydrocarbons through pipelines, liquid-vapor flow in power-plants, etc (Dukler e Fabre, 1992). The slug is a flow pattern which is highly intermittent, it is formed by sequences of large gas bubbles followed by packs of liquid (slug) flowing in random fluctuating frequencies (Fabre e Liné, 1992; Woods et al., 2000).

The slug pattern can be formed in horizontal and inclined pipelines from a stratified pattern by basically two mechanisms: the natural growth of hydrodynamic instabilities and by the accumulation of liquid due to irregularities on the terrain. In the first case, small perturbations in the form of small waves naturally emerge. These waves can grow to larger waves of the size of the pipeline cross-section (Ansari, 1998). The growth mechanism is the classic Kelvin-Helmholtz instability (KH) (Lin e Hanratty, 1986; Fan et al., 1993a,b). These waves can continue to grow, capturing the liquid that flows in front of them until the cross section becomes saturated with liquid, thus forming the *slugs*. At inclined pipelines, the slug can be formed due to the delay and sub-sequent accumulation of liquid at the down points of the pipeline, leading to a cross section completely filled with liquid. This type of slug pattern induced by the terrain is called "*severe slugging*" and it can be formed when a slightly inclined pipeline joins a riser, that is, a vertical pipeline (Jansen et al., 1996). The flow in the slug pattern can also be formed by a combination of the mechanisms described above. Small undulations of the terrain can lead to slug formation in addition to the ones formed by the inherent instabilities of the flow. In theses cases, the slug formed

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by one mechanism interacts with those formed by the other, leading to a complex slug pattern.

The intermittence of the flow in the slug pattern causes large instabilities, which propagates through out the pipeline and any other equipment connected to it. This often increases the design problems and it usually leads to a reduction of the efficiency and/or size of a processing plant. Thus, it is important to be able to predict the beginning and subsequent development of the slug pattern, as well as the prediction of its characteristics such as size and frequency.

There are several methods to predict the slug formation, among them, the "*slug capturing*" methodology can be mentioned, which is based on the one-dimensional two fluid model (Ishii,1975), to predict two-phase flows. In this model, each phase is described by a set of mass, momentum and energy conservation equations, which are obtained by an average process for each phase. The "*slug capturing*" methodology was employed by Issa e Kempf (2003), to predict the transition of a stratified flow to a slug pattern, and by Oliveira e Issa (2003) who investigated numerical aspects of the solution of the two-fluid model, presenting methodologies to limit the volume fraction within physical limits. In 2003, Bonnizi e Issa presented two papers employing the *"slug capturing"* model. At the first one, the slug pattern for a three phase fluid was obtained with the multi-fluid model, while the second one concerns the entrainment of small bubbles of gas in the liquid.

The objective of the present work is to investigate the capacity of the two-fluid model to predict of natural appearance and subsequent development of the slug pattern from the stratified pattern.

2. Mathematical Modeling

The mathematical model selected is based on the "*slug capturing*" technique, in which the slug formation is predicted as a result of a natural and automatic growth of the hydrodynamic instabilities (Issa e Kempf, 2003). Both stratified and slug pattern are modeled by the same set of conservation equations based on the two-fluid model. Additionally, closure relations are included. The first hypothesis to be mentioned consists on assuming a uniform pressure *P* along the cross section, i.e. same pressure for the liquid, gas and interface. The liquid is considered as incompressible, while the gas follows the ideal gas law,

$$\rho_g = P/(RT) \tag{1}$$

where *R* is the gas constant and *T* is its temperature, which was considered here as constant.

The governing mass and momentum equations for each phase can be written as:

$$\frac{\partial(\rho_g\ \alpha_g)}{\partial t} + \frac{\partial(\rho_g\ \alpha_g\ u_g)}{\partial x} = 0\,, \tag{2}$$

$$\frac{\partial(\rho_\ell\ \alpha_\ell)}{\partial t} + \frac{\partial(\rho_\ell\ \alpha_\ell\ u_\ell)}{\partial x} = 0\,, \tag{3}$$

$$\frac{\partial(\rho_g \alpha_g u_g)}{\partial t} + \frac{\partial(\rho_g \alpha_g u_g^2)}{\partial x} = -\alpha_g \frac{\partial P}{\partial x} - \rho_g \alpha_g g\,\mathbf{sen}(\beta) - \rho_g \alpha_g g \frac{\partial h}{\partial x}\mathbf{cos}(\beta) - F_{gw} - F_i\,, \tag{4}$$

$$\frac{\partial(\rho_\ell \alpha_\ell u_\ell)}{\partial t} + \frac{\partial(\rho_\ell \alpha_\ell u_\ell^2)}{\partial x} = -\alpha_\ell \frac{\partial P}{\partial x} - \rho_\ell \alpha_\ell g\,\mathbf{sen}(\beta) - \rho_\ell \alpha_\ell g \frac{\partial h}{\partial x}\mathbf{cos}(\beta) - F_{\ell w} + F_i\,, \tag{5}$$

where $\alpha_g+\alpha_l=1$. The subscripts *g*, *l*, and *i* concern the gas, liquid phases and interface, respectively. The axial coordinate is *x*, ρ and α are the density and volumetric fraction, *u* is the velocity. The pipeline inclination is β, *h* is the liquid level inside the pipe, and *g* is the gravity acceleration. The third term on the right side of Eqs. (4) and (5) are related with the hydrostatic pressure at the gas and liquid, respectively. The term $F = \tau\ S\ /\ A$ is the friction force per unit volume between each phase and the wall and between the phases (at the interface), where τ is the shear stress, *S* is the phase perimeter and *A* is the pipe cross section area.

The shear stress is $\tau = f\,\rho\,|\ u_r|\ u_r\ /\ 2$, where u_r is the relative velocity between the liquid and wall, the gas and wall, or gas and liquid. Closure relations are needed to determine the friction factor *f*.

The flow was considered in the laminar regime, when the Reynolds number Re, was smaller the 2100 (Re_g ; Re_i and Re_l for the gas, interface and liquid, respectively). The Hagen-Poisseulle formulas were employed for the gas-wall and interface friction factor and the correlation of Hand (1991) for the liquid-wall friction factor:

$$f_g = 16 / \mathbf{Re}_g, \qquad f_i = 16 / \mathbf{Re}_i, \qquad f_l = 24 / \mathbf{Re}_l^s, \tag{6}$$

while the Taitel e Dukler (1976) was adopted for the turbulent gas-wall and interface friction factor and the Spedding and Hand (1997) correlation for turbulent the liquid-wall friction

$$f_g = 0.046 \left(\mathbf{Re}_g\right)^{-0.25}, \qquad f_i = 0.046 \left(\mathbf{Re}_i\right)^{-0.25}, \qquad f_l = 0.0262 \left(\alpha_l \, \mathbf{Re}_l^s\right)^{-0.139} \tag{7}$$

where α_l is the *holdup* (liquid volumetric fraction). The Reynolds numbers are defined as

$$\mathbf{Re}_g = \frac{4 A_g u_g \rho_g}{\left(S_g + S_i\right) \mu_g}, \quad \mathbf{Re}_i = \frac{4 A_g \left|u_g - u_l\right| \rho_g}{\left(S_g + S_i\right) \mu_g}, \quad \mathbf{Re}_l = \frac{4 A_l \, u_l \, \rho_l}{S_l \mu_l}, \quad \mathbf{Re}_l^s = \frac{\rho_l U s_l D}{\mu_l}. \tag{8}$$

where μ is the absolute viscosity and D is the pipe diameter. The last Reynolds in Eq. (8) is based on the liquid superficial velocity, i.e., the ration of the liquid volume flow rate to the total cross section area of the pipe:

$$U s_l = \frac{Q_l}{A} \qquad \text{or} \qquad U s_l = \alpha_l \; u_l \tag{9}$$

When a slug is formed, the liquid volume fraction becomes one and the gas volume fraction goes to zero. As a result, the gas momentum equation, Eq. (4), becomes singular, since α_g appears in both sides of the equation.

Figure 1 presents a sketch of the pipeline, with a cross section area A and diameter D. The areas and wetted perimeters of the gas and liquid are A_g, S_g and A_ℓ, S_ℓ, respectively, and the interface width is S_i. All these geometric parameters depend only on liquid height h (Spedding e Hand, 1997):

$$S_g = D \cos^{-1}(\xi) \quad ; \quad S_l = \pi \, D - S_g \quad ; \quad S_i = D \sqrt{1 - \xi^2} \tag{10}$$

$$A_g = \frac{D}{4} \left[S_g - \xi \; S_i \right] \quad ; \quad A_l = \frac{\pi \, D^2}{4} - A_g \quad ; \quad \xi = 2 \; (D \, / \, h) - 1 \tag{11}$$

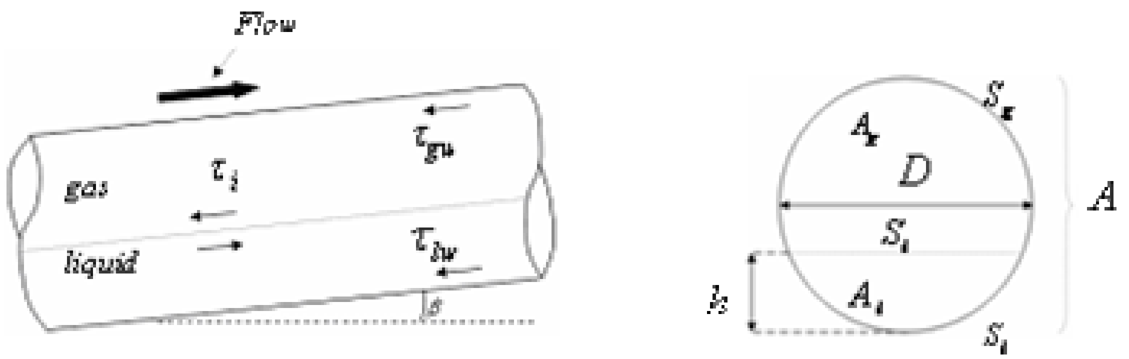

Figure 1. Sketch of the pipe, and its cross section

Further, the liquid height is a geometric function of the gas volumetric fraction ($\alpha_g = A_g \; / \; A$), and can be obtained by the solution of

$$\alpha_g = \frac{1}{\pi} [\cos^{-1}(\xi) - \xi \; \sqrt{1 - \xi^2} \,] \tag{12}$$

3. Numerical Method

The conservation equations were discretized by the Finite Volume Method (Patankar, 1980). A staggered mesh was employed, with both phases' velocities stores at the control volume faces and all other variables at the central point. The interpolation scheme *Upwind* and the implicit *Euler* scheme were selected to evaluate the space and time derivates, respectively.

A conservative approach was selected to discretize the conservation equations, based on the analysis of Ortega Malca and Nieckele (2005), who investigated the advantages/disadvantages of employing a conservative or non-conservative

The set of resulting equations consists of two momentum equations, one pressure equation (global mass conservation) and one gas volumetric fraction (gas mass conservation). These equations were solved sequentially, through an iterative method. Details can be found in Ortega Malca and Nieckele (2005).

Since the gas momentum equation becomes singular when the gas volumetric fraction becomes zero, this equation was not solved when a slug was formed (α_g<0.02), and the gas velocity was arbitrarily set to zero, as recommended by Issa e Kempt (2003) and Bonizzi & Isaa (2003a).

For each time step, due to the non linearities of the problem, the sequence of conservation equations were solved in an iterative process, until convergence was obtained, that is, until the residue of all equations became smaller than 0.0001.

4. Results

Figure 2 illustrates the problem configuration. It is exactly the same as the one investigated by Issa & Kempf (2003) and Bonizzi (2003a). The pipeline is horizontal, with length equal to L=36 m, and internal diameter D = 0.078m. The flow was considered isothermal at a reference temperature T = 281.15 K. The outlet pressure P_{out} was kept constant equal to the atmospheric pressure. The gas phase is air (gas constant, R = 287 N m /(Kg K) and the absolute viscosity is μ = 1.796× 10^{-5} Pa-s) and the liquid phase water (density, ρ = 998.2 kg/m^3, absolute viscosity, μ = 1.139 × 10^{-3} Pa-s). At the inlet the superficial liquid and gas superficial velocities, U_{sl} and U_{sg} , were specified as constant, as well the liquid *holdup* α_l

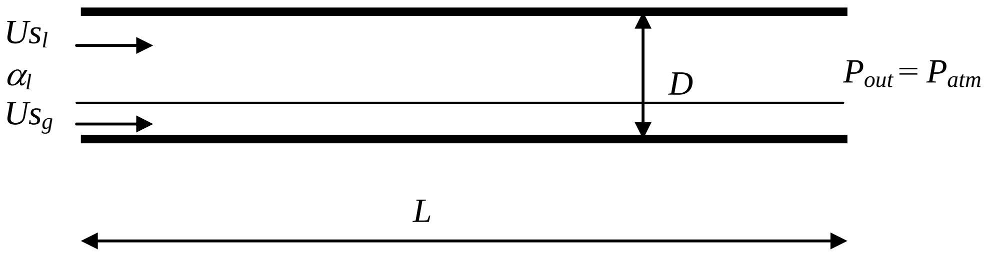

Figure 2. Problem test configuration.

The initial condition was defined as a stratified steady state flow, that is, constant liquid height along the pipeline, with constant liquid and gas velocities. The pressure distribution was obtained by integrating the following equation, resulting from a combination of the momentum equation of the liquid and gas phase, for the equilibrium stratified flow

$$\partial P \;/\; \partial x \;=\; -(\tau_{lw} S_l \;+\; \tau_{gw} S_g) \;/\; A \tag{13}$$

The liquid volume fraction for an equilibrium stratified flow, can be determined as function of superficial velocities, Eq.(9), by combining the momentum equation for both phases and elimination the pressure gradient, resulting in the following expression

$$\left(\rho_l - \rho_g\right) g \,\mathbf{sen(}\beta\mathbf{)} + \frac{\tau_{lw} S_l}{\alpha_l A} - \frac{\tau_{wg} S_g}{\alpha_g A} - \frac{\tau_i S_i}{A}\left(\frac{1}{\alpha_l} + \frac{1}{\alpha_g}\right) = 0 \tag{14}$$

To be able to predict the slug pattern, it is necessary a well posed system of equations. Additionally, the transition from the stratified to the slug pattern can only happen if the boundary conditions induce an unstable flow.

Therefore, a Kelvin-Helmoltz stability analysis was performed (Taitel & Dukler, 1976) aiming the selection of the boundary conditions (U_{sl} ,U_{sg} , α_l) suitable for the present problem.

A map of flow pattern (Taitel & Dukler, 1976) can be seen in Fig. 3, for the pipeline configuration adopted at the present work. This map shows the range of gas and liquid superficial velocities which leads to the different flow patterns (bubbles, slug, annular, stratified).

Based on the analysis presented by Bonzini & Issa (2003b), at the same map, an additional curve was plotted, separating the regions where the system of equations is well and ill posed. Chum & Sung, (1996) have shown that for an incompressible flow, the two fluid model is well posed when the relative phase velocities are smaller than the Kelvin-Helmolts stability criteria for non viscous flow, as developed by Barnea & Taitel (1994). Bonizzi (2003) showed that the same criteria can be applied even when the gaseous phase is compressible.

$$\left| u_g - u_l \right| < \sqrt{\left(\rho_l \, \alpha_g + \rho_g \, \alpha_l\right) \frac{\rho_l - \rho_g}{\rho_l \rho_g} \, g \, \mathbf{cos}(\beta) \frac{A}{S_i}} \tag{15}$$

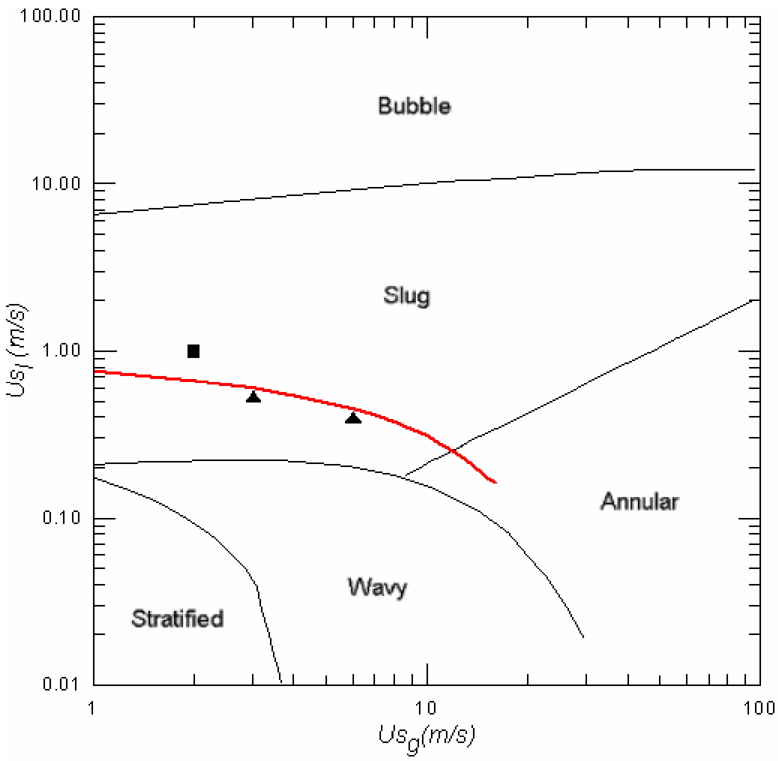

Figure 3. Flow pattern map of Taitel & Dukler (1976). Location of test cases on the map.

Three cases were selected to be presented here. These cases are also indicated at the map of Fig. 3. The selected test cases are:

Test 1: Us_g = 2.0 m/s; Us_l = 1.0 m/s and α_l = 0.4.

Test 2: Us_g = 3.0 m/s; Us_l = 0.55 m/s and liquid volume fraction corresponding to the stratified equilibrium condition α_l^{eq} = 0.743.

Test 3: Us_g = 6.0 m/s, Us_l = 0.4 m/s and liquid volume fraction corresponding to the stratified equilibrium condition α_l^{eq} = 0.566.

The first case selected, employed the boundary condition utilized by Issa & Kempf (2003). It can be seen in Fig. 3 that its superficial velocities are above the line that defines the regions of well and ill posed solutions for the problem; therefore it should be ill posed. For the second and third test cases, the boundary conditions were specified based on the work of Bonnizi & Issa (2003a), and the superficial velocities are in the well posed region.

For all cases, a uniform mesh was specified in the domain, with 1250 nodes (Δx/D=0.369) as employed by Bonizzi & Issa (2003a). The time step was specified to guarantee a Courant number equal to 0.5 (Issa e Kempf, 2003), therefore, the time step was obtained from $\Delta t = 0.5 \; \Delta x_i \, / \left| \, u_{\max} \right|$, where u_{max} is the maximum velocity in the domain.

Figure 4 illustrates for the first case, the liquid *holdup* evolution along the pipeline, for different time instants (each curve corresponds to a different time), while Figs. 5 and 6 correspond to cases 2 and 3 respectively.

Ortega Malca & Nieckele (2005) presented a comparison of the first case with the results of Issa & Kempt (2003), where a reasonable agreement was obtained. Although the superficial velocities are in the ill posed region, the slug was formed and displaced along the pipe.

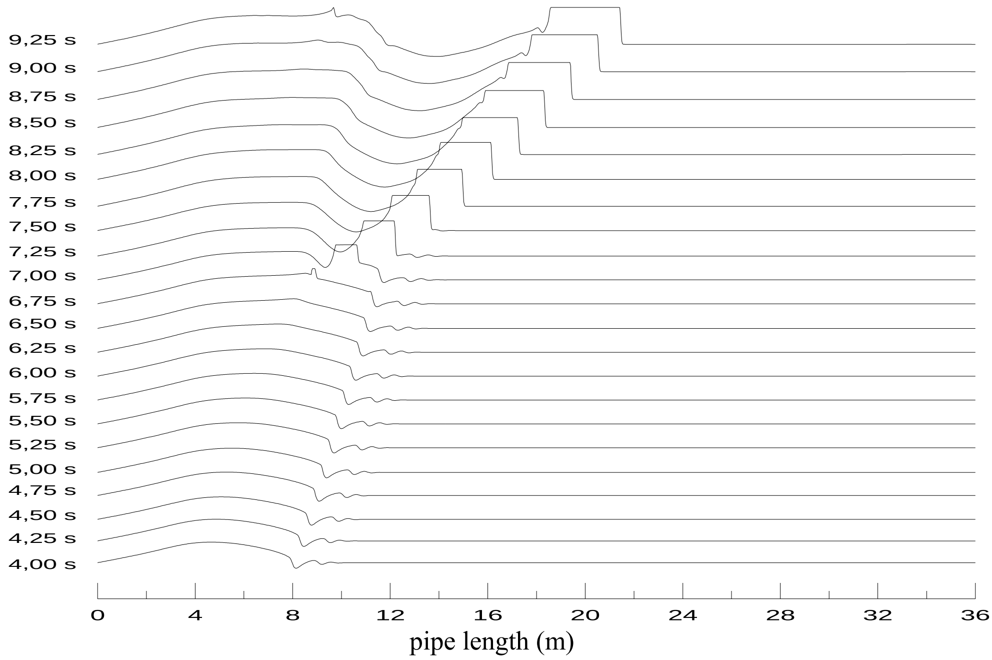

Figure 4 - Liquid *holdup* distribution along the pipe for different time instants. $Us_g = 2.0$ m/s e $Us_l = 1.0$ m/s.

For the other two cases, the formation of the slug, their growths, as well as their displacement along the pipeline are shown in Figs. 5 and 6. It can be seen that the slug length is larger for case 2 than case 3. It can also be seen that the first slug appears earlier for case 2.

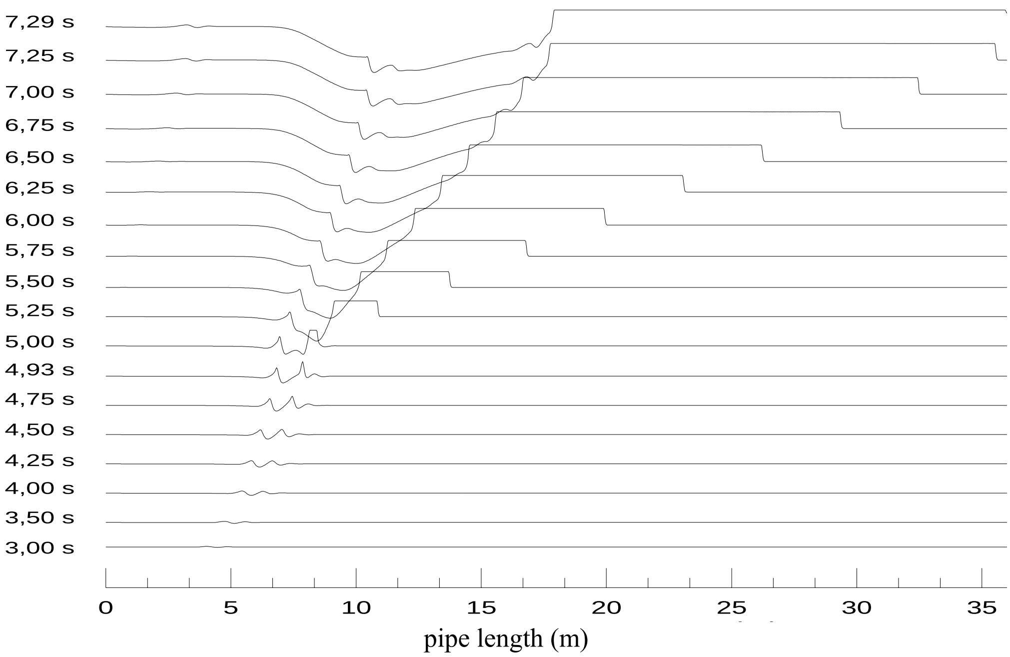

Figure 5. Liquid *holdup* distribution along the pipe for different time instants. $Us_g = 3.0$ m/s e $Us_l = 0.55$ m/s.

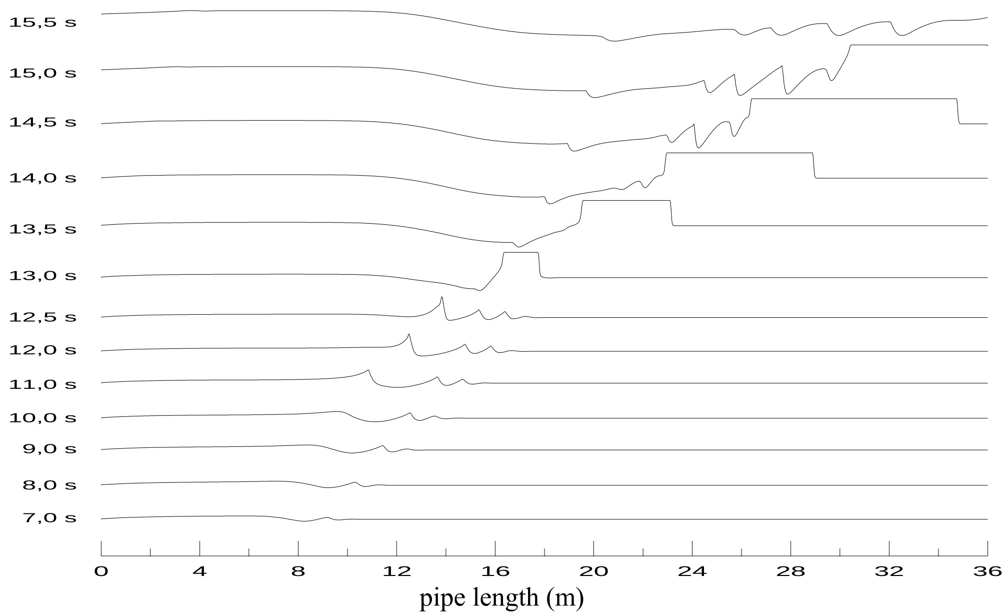

Figure 6. Liquid *holdup* distribution along the pipe for different time instants. Us_g = 6.0 m/s e Us_l = 0.4 m/s.

By examining Figs. 4 to 6 it is clear the influence of the operating conditions. The time instance of appearance of the first slug, as well as its length, frequency and translational velocity depend on the superficial velocities at the entrance and on the inlet liquid *holdup*.

5. Final Remarks

Depending on the superficial velocity ratio, different flow patterns can be found. Further, due to physical instabilities, transition from one flow pattern to another occurs. These instabilities by themselves can introduce difficulties in predicting these types of flows. Additionally numerical instabilities are also encountered. Therefore, the prediction of slug flow is highly complex, due to the presence of both physical and numerical instabilities.

At the present work the one-dimensional two-fluid model as described by Issa & Kempf (2003) and Bonizzi & Issa (2003a), was employed to predict the transition from the stratified flow pattern to the slug pattern. The methodology developed was able to predict the slug formation, growth and displacement along the pipeline.

6. Acknowledgement

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7. References

ANSARI, M. R. Dynamical behavior of slug initiation generated by short waves in two-phase air-water stratified flow. *ASME HTD 361*, pp. 289-295, 1998.

BARNEA, D., TAITEL, Y. Interfacial and structural stability of separated flow. Int. J. Multiphase Flow 20, 387–41., 1994.

BONIZZI, M.; ISSA, R.I. A model for simulating gas bubble entrainment in two-phase horizontal slug flow. *International Journal of Multiphase Flow*, Vol. 29, pp.1685-1717, 2003a.

BONIZZI, M.; ISSA, R.I. On the simulation of three-phase slug flow in nearly horizontal pipes using the multi-fluid model. *International Journal of Multiphase Flow*, Vol. 29, pp.1719-1747, 2003b.

CHUN, M. H. & SUNG, C. K. Onset of slugging criterion based on characteristics and stability analyses of transient one-dimensional two-phase flow equations of the two fluid model. Int. Comm. Heat Mass Transfer 23, 473-484., 1996.

DUKLER, A.E.; FABRE, J. Gas–liquid slug flow-knots and loose ends. *Proceedings of 3rd International Workshop Two-Phase Flow Fundamentals*, Imperial College, 1992.

FABRE, J.; LINÉ, A. Modeling of two-phase slug flow. *Ann. Rev. Fluid Mech.* Vol. 24, pp. 21–46, 1992.

FAN, Z.; LUSSEYRAN, F.; HANRATTY, T.J. Initiation of slugs in horizontal gas-liquid flows. *AIChE Journal*, Vol. 39, pp. 1741-175, 1993a.

HAND, N.P. *Gas–liquid co-current flow in a horizontal pipe*. Ph.D. Thesis, Queen`s University Belfast, 1991.

ISHII, M., 1975. Thermo-Fluid Dynamic Theory of Two-Phase Flow. Eyrolles, Paris, 1975.

ISSA, R.I.; KEMPF, M.H.W. Simulation of slug flow in horizontal and nearly horizontal pipes with the two-fluid model. *International Journal Multiphase Flow*, Vol. 29, pp. 69-95, 2003.

JANSEN, F.E.; SHOHAM, O.; TAITEL, Y. The elimination of severe slugging. *International Journal Multiphase Flow*, Vol. 22, pp. 1055–1072, 1996.

LIN, P.Y.; HANRATTY, T.J. Prediction of the initiation of slugs with linear stability theory. *International Journal Multiphase Flow*, Vol. 12, pp. 79-98, 1986.

PATANKAR, 1980, *Numerical Heat Transfer and Fluid Flow*, Hemisphere Publishing Corporation.

OLIVERA, P.J.; ISSA, R.I. Numerical aspects of an algorithm for the Eulerian simulation of two-phase flow. *International Journal of Numerical Methods in Fluids*, Vol. 43, pp. 1177-1198, 2003.

ORTEGA MALCA, A.; NIECKELE, A.O. Simulation of horizontal two-phase slug flows using the two-fluid model with a conservative and non-conservative model. *18th Int. Congress of Mechanical Engineering – COBEM 2005*, 2005.

SPEDDING, P.L.; HAND, N.P. Prediction in stratified gas–liquid co-current flow in horizontal pipelines. *Int. J. Heat Mass Transfer* 40, 1923–1935, 1997.

TAITEL, Y.; DUKLER, A.E. A model for predicting flow regime transitions in horizontal and near horizontal gas–liquid flow. *AIChE Journal*, Vol. 22, pp. 47–55, 1976.

WOODS, B.D.; HURLBURT, E.T.; HANRATTY, T.J. Mechanism of slug formation in downwardly inclined pipes. *International Journal Multiphase Flow*, Vol. 26, pp. 977–998, 2000.